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(1) Test the convergent of series

(A) $\sum_{n=1}^{\infty}\left[\frac{2}{(-1)^n-3}\right]^n$ (B) $\sum e^{-n^2}$ (C) $\left\{\frac{\log n}{\log(n+1)}\right\}^{n^2 \log n}$

(2) Give an example of seq^n $\langle a_n\rangle$ and $\langle b_n\rangle$ s.that both seq^n converge to zero as $n\to\infty$ and the seq^n $c_n = \frac{(a_n)^2}{b_n}$ converges to 5.

(3) Let $\{a_n : n\geq 1\}$ be a seq^n s.that $a_n \geq 0$ for $\forall n\geq 1$ which of the following sets of condition implies that the seq^n $\{a_n : n\geq 1\}$ is convergent

(A) The subseq^n $\{a_{2n} ; n\geq 1\}$, $\{a^2_{3n} ; n\geq 1\}$ and $\{a^3_{2n+1} ; n\geq 1\}$ are convergent.

(B) The subseq^n $\{a_{3n} ; n\geq 1\}$, $\{a^2_{5n}\}$ $\{a^3_{2n+1}\}$ are cgt

(C) The subseq^n $\{a_{5n} ; n\geq 1\}$, $\{a^2_{7n}\}$ $\{a^3_{2n+1}\}$ are cgt

(D) The subseq^n $\{a_{7n} ; n\geq 1\}$, $\{a^2_{2n}\}$ $\{a^3_{2n+1}\}$ are cgt.

(4) Let $\{a_n ; n\geq 1\}$ be a seq^n of real numbers, $S_n = \sum_{k=1}^{n} a_k$ and $m_n = \frac{S_n}{n}$, which of the following

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(A) If $\{s_n\}$ converges to a real number s, then $\{a_n\}$ converges to 0.

(B) If $\{a_n\}$ converges to 0 then $\{s_n\}$ converges to a real number s.

(C) If $\{m_n\}$ converges to a real number m, then $\{a_n\}$ converges to m.

(d) If $\{a_n\}$ converges to a real number a, then m_n converges a.

(5) Let $a_n = \frac{b_{n+1}}{b_n}$ where $b_1 = 1$, $b_2 = 1$ and $b_{n+2} = b_n + b_{n+1}$; $n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is

B $\frac{1+\sqrt{5}}{2}$ Proved that.

(6) Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and $c_n = \sum_{k=0}^{n} a_{n-k} a_k$ where $n \in \mathbb{N} \cup \{0\}$. Then which one of the following is True

(A) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are cgt.

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(b) $\sum_{n=0}^{\infty} a_n$ is cgt but $\sum_{n=1}^{\infty} a_n$ is not cgt.

(c) $\sum_{n=1}^{\infty} a_n$ is cgt but $\sum_{n=0}^{\infty} a_n$ is not cgt.

(7) Let $a_k = (-1)^{k-1}$, $S_n = a_1 + a_2 + \cdots + a_n$ and

$$\sigma_n = \frac{S_1 + S_2 + \cdots + S_n}{n}$$ where $k, n \in \mathbb{N}$, Then

$\lim_{n\to\infty} a_n$ is ______.

(8) Let $a_n = \sqrt{n}$, $n \geq 1$ and let $S_n = a_1 + a_2 + \cdots + a$

Then $$\lim_{n\to\infty}\left(\frac{a_n/S_n}{-\ln\left(1 - \frac{a_n}{S_n}\right)}\right) = \text{______}.$$

(9) Let a_n be a seq^n of strictly +ve numbers

s. that $\lim_{n\to\infty} \frac{a_n}{a_{n+2}} = 2$ then Radius of cgt

(A) of the series $\sum a_n x^n$ is $\frac{1}{2}$

(B) of the series $\sum \frac{1}{a_n} x^n$ is $\frac{1}{2}$

(C) " $\sum a_n^2 x^n$ is 4

(D) " $\sum (a_n)^n x^n$ is ∞.

[illegible] Suppose that the power series $f(x) = \sum b_n x^n$ (converges for $|x| < 1$. Suppose that for some $\delta > 0$, $f(x) = 0$ for $|x| < \delta$. Show that $b_n = 0$ for $\forall\, n \geq 1$

[illegible] $a_n = \frac{b_{n+1}}{b_n}$ where $b_1 = 1$, $b_2 = 1$

$$b_{n+2} = b_n + b_{n+1} \quad n \in \mathbb{N}$$

Then $\lim_{n \to \infty} a_n$ is ______ ?

some

$b_n = 0$

VED PRAKASH

→ Doubts

Date

(1) Let $\sum a_n$ be a convergent series of non-negative terms. Then

(A) $\liminf_n n a_n = 0$ (B) $\limsup_n n a_n = 1$

(C) $0 < \liminf_n n a_n < 1$ (D) $\limsup_n a_n > 0$.

(2) Suppose that $f: \mathbb{R} \to \mathbb{R}$ is cts and $|f(x) - f(y)| \geq \frac{3}{4}|x-y|$ $\forall$ x, y in $\mathbb{R}$. Then $f(\mathbb{R})$ is

(A) $\mathbb{R}$ (B) $\emptyset$ (C) an interval not necessarily $\mathbb{R}$ (D) $[0, \infty)$

(3) If
$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Is $f(x,y)$ cts on $\mathbb{R}^2$. Find $f_x(x,y)$, $f_x(0,0)$, $f_y(0,0)$, $f_{xx}(0,0)$, $f_{yy}(0,0)$, $f_{xy}(0,0)$, $f_{yx}(0,0)$.

2nd (4) Discuss questions based on Improper Integral.

(5) Discuss some question of differentiability in function of several variables.

(6) Last test Discussion.

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Real Analysis problems

Nilkantha Das.

1st Q. Consider the following functions

$$f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1, & |x| \le 2 \\ 2, & |x| > 2 \end{cases}$$

Define $h_1(x) = f(g(x))$ and $h_2(x) = g(f(x))$. Which of the following statements is correct?

(A) h_1 and h_2 are continuous everywhere.

(B) h_1 is continuous everywhere and h_2 has discontinuity at ± 1.

(C) h_2 is continuous everywhere and h_1 has discontinuity at ± 2.

(D) h_1 has discontinuity at ± 2 and h_2 has discontinuity at ± 1.

2nd Q. Number of real roots of the equation

$$x^7 + 2x^5 + 3x^3 + 4x = 2018$$

(A) 1 (B) 3 (C) 5 (D) 7.

~~3rd Q.~~

~~Let $a, b \in \mathbb{R}$ such that~~

~~$\lim_{x \to 0} \left(\frac{\sin 3x}{\ } \right.$~~

~~A~~ sequence of real numbers x_n are defined as follows

$x_{n+2} = \frac{1 + x_{n+1}}{x_n}$, $n = 0, 1, 2, \dots$ and

$x_0 = 1$, $x_1 = 2$.

Then x_{2014} equals to

(A) 1 (B) 2 (C) 3 (D) None of the above.

4th Q.

Let $S = \{1, 2, \dots, 10\}$. Then the number of pairs (A, B), where A and B are non-empty disjoint subsets of S is

(A) $3^{10} - 1$ (B) $3^{10} - 2^{10}$ (C) $3^{10} - 2^{10} + 1$ (D) $3^{10} - 2^{11} + 1$.

5th Q.

Let $f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n} \sin x}{1 + x^{2n}}$, $x > 0$

Then

(A) f is continuous at $x = 1$.

(B) $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$

(C) $\lim_{x \to 1^+} f(x) = \sin 1$.

(D) $\lim_{x \to 1^-} f(x)$ does not exist.

Let $f(x) = x^{\pi} + \frac{1}{x^{\pi}} + x + \frac{1}{x}$, $x > 0$

and let $m = \min\{f(x)\}$. Then

(A) $m = 1$ (B) $m = 4$, (C) $m = \frac{27}{4}$, (D) m does not exist.

7th Q.

The integral $\int_0^1 \frac{\sin x}{x^{\alpha}}\,dx$

(A) is finite only for $\alpha = 0$.

(B) is finite only for $|\alpha| < 1$.

(C) is finite for all $\alpha < 2$.

(d) is infinite for any value of α. 1

8th Q. The integral $\int_0^{\infty}\int_0^{\infty} \frac{e^{-(x+y)}}{(x+y)}\,dx\,dy$

is

(A) infinite

(B) finite, but cannot be evaluated in closed form.

(C) 1 (D) 2.

9th Q. The number of real roots of the eqn

$2\cos\left(\frac{x^2+x}{6}\right) = 2^x + 2^{-x}$ is

(A) 0 (B) 1 (C) 2 (D) ∞.

$k=1$ ... $n = \frac{a_n}{n}$, which of the follo...

(29) Q. **10th Q.** Suppose a is real number for which all the roots of the eqn

$x^4 - 2ax^2 + x + a^2 - a = 0$ are real. Then

(A) $a < -2/3$ (B) $a = 0$ (C) $0 < a < 3/4$ (d) $a \geq 3/4$.

11th Q. Consider the functions

$$f(x) = \frac{e^{-|x|}}{\max\{e^x, e^{-x}\}}, \quad x \in \mathbb{R}$$

(A) f is not continuous at some points.

(B) f is continuous everywhere, but not differentiable anywhere.

(C) f is continuous everywhere, but not differentiable at exactly one point.

(D) f is differentiable everywhere.

12th Q. Given that $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$, what is the value of

$$\int_{-\infty}^{\infty} |x|^{-1/2} e^{-|x|}\,dx\,?$$

(A) 0 (B) $\sqrt{\pi}$ (C) $2\sqrt{\pi}$ (D) ∞.

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